

## Thermodynamic Processes (Derivations)

### 1 Work Done in Isothermal Process

For an isothermal process (T fixed), the ideal gas equation gives

$$PV = \text{constant} \quad (1)$$

Suppose an ideal gas goes isothermally (at temperature T) from its initial state ( $P_1, V_1$ ) to the final state ( $P_2, V_2$ ). At any intermediate stage with pressure P and volume change from V to  $V + \Delta V$  ( $\Delta V$  small)

$$\Delta W = P\Delta V \quad (2)$$

Taking ( $\Delta V \rightarrow 0$ ) and summing the quantity  $\Delta W$  over the entire process,

$$W = \int_{V_1}^{V_2} P dV \quad (3)$$

$$= \mu RT \int_{V_1}^{V_2} \frac{dV}{V} \quad (4)$$

$$\boxed{W = \mu RT \ln \frac{V_2}{V_1}} \quad (5)$$

### 2 Work Done in Adiabatic Process

For an adiabatic process of an ideal gas

$$PV^\gamma = \text{constant} \quad (6)$$

Where,

$$\gamma = \frac{C_p}{C_v} \quad (7)$$

Thus if an ideal gas undergoes a change in its state adiabatically from ( $P_1, V_1$ ) to ( $P_2, V_2$ ):

$$P_1 V_1^\gamma = P_2 V_2^\gamma \quad (8)$$

The work done in an adiabatic change of an ideal gas from the state ( $P_1, V_1, T_1$ ) to the state ( $P_2, V_2, T_2$ ).

$$W = \int_{V_1}^{V_2} P dV \quad (9)$$

$$\begin{aligned} &= \text{constant} \times \int_{V_1}^{V_2} \frac{dV}{V^\gamma} = \text{constant} \times \left[ \frac{V^{-\gamma+1}}{1-\gamma} \right]_{V_1}^{V_2} \\ &= \frac{\text{constant}}{1-\gamma} \times \left[ \frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}} \right] \end{aligned} \quad (10)$$

From Eqn (8), the constant is  $P_1 V_1^\gamma$  or  $P_2 V_2^\gamma$

$$W = \frac{1}{1-\gamma} \left[ \frac{P_2 V_2^\gamma}{V_2^{\gamma-1}} - \frac{P_1 V_1^\gamma}{V_1^{\gamma-1}} \right] \quad (11)$$

$$= \frac{1}{1-\gamma} [P_2 V_2 - P_1 V_1]$$

$$W = \frac{\mu R(T_1 - T_2)}{\gamma - 1} \quad (12)$$

- If work is done **by** the gas in an adiabatic process ( $W > 0$ ), from Eq. (12),  $T_2 < T_1$ .
- If work is done **on** the gas ( $W < 0$ ), we get  $T_2 > T_1$  i.e., the temperature of the gas rises.

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