Thermodynamic Processes (Derivations)

1 Work Done in Isothermal Process

For an isothermal process (T fixed), the ideal gas equation gives

$$PV = constant \tag{1}$$

Suppose an ideal gas goes isothermally (at temperature T) from its initial state (P_1, V_1) to the final state (P_2, V_2). At any intermediate stage with pressure P and volume change from V to $V + \Delta V (\Delta V$ small)

$$\Delta W = P\Delta V \tag{2}$$

Taking $(\Delta V \to 0)$ and summing the quantity ΔW over the entire process,

$$W = \int_{V_1}^{V_2} P dV \tag{3}$$

$$= \mu RT \int_{V_1}^{V_2} \frac{dV}{V} \tag{4}$$

$$\boxed{W = \mu RT \ln \frac{V_2}{V_1}} \tag{5}$$

2 Work Done in Adiabatic Process

For an adiabatic process of an ideal gas

$$PV^\gamma = constant \tag{6}$$

Where,

$$\gamma = \frac{C_p}{C_v} \tag{7}$$

Thus if an ideal gas undergoes a change in its state adiabatically from (P_1, V_1) to (P_2, V_2) :

$$P_1 V_1^\gamma = P_2 V_2^\gamma \tag{8}$$

The work done in an adiabatic change of an ideal gas from the state (P_1, V_1, T_1) to the state (P_2, V_2, T_2).

$$W = \int_{V_1}^{V_2} P dV \tag{9}$$

$$= constant \times \int_{V_1}^{V_2} \frac{dV}{V^\gamma} = constant \times \left[\frac{V^{-\gamma+1}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{constant}{1-\gamma} \times \left[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}}\right] \tag{10}$$

From Eqn (8), the constant is $P_1 V_1^\gamma$ or $P_2 V_2^\gamma$

$$W = \frac{1}{1-\gamma}\left[\frac{P_2 V_2^\gamma}{V_2^{\gamma-1}} - \frac{P_1 V_1^\gamma}{V_1^{\gamma-1}}\right] \tag{11}$$

$$= \frac{1}{1-\gamma}\left[P_2 V_2 - P_1 V_1\right]$$

$$\boxed{W = \frac{\mu R(T_1 - T_2)}{\gamma - 1}} \tag{12}$$

- If work is done **by** the gas in an adiabatic process ($W > 0$), from Eq. (12), $T_2 < T_1$.
- If work is done **on** the gas ($W < 0$), we get $T_2 > T_1$ i.e., the temperature of the gas rises.

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