Kendriya Vidyalaya, Masjid Moth, Sadiq Nagar, New Delhi

Chishant Nimesh, PGT Physics

Motion in a Straight Line

1 FRAME OF REFERENCE AND POSITION MEASUREMENT

1.1 Frame of Reference

- A rectangular coordinate system consisting of three mutually perpendicular axes, labeled X-, Y-, and Z- axes.
- The point of intersection of these three axes is called **origin (O)** and serves as the **reference point**.
- The coordinates (x, y. z) of an object describe the position of the object with respect to this coordinate system.
- The coordinate system along with a time measurement is called **frame of reference**.

1.2 Distance

The **distance** is the path length covered by the object.

1.3 Displacement

- Displacement has both magnitude and direction. it is a vector quantity.
- In one-dimensional motion, there are only two directions (backward and forward,upward and downward) in which an object can move, and these two directions can easily be specified by + and - signs.
- The magnitude of displacement may or may not be equal to the path length traversed by an object.
- **Displacement**, denoted by Δx, in time $\Delta t = (t_2 - t_1)$, is given by the difference between the final and initial positions :

$$\Delta x = x_2 - x_1 \tag{1.1}$$

  Where, Greek letter delta (Δ) denotes a change in a quantity.

2 POSITION-TIME GRAPH

- Motion of an object can be represented by a position-time graph.
- If an object moving along the straight line covers equal distances in equal intervals of time, it is said to be in uniform motion along a straight line.

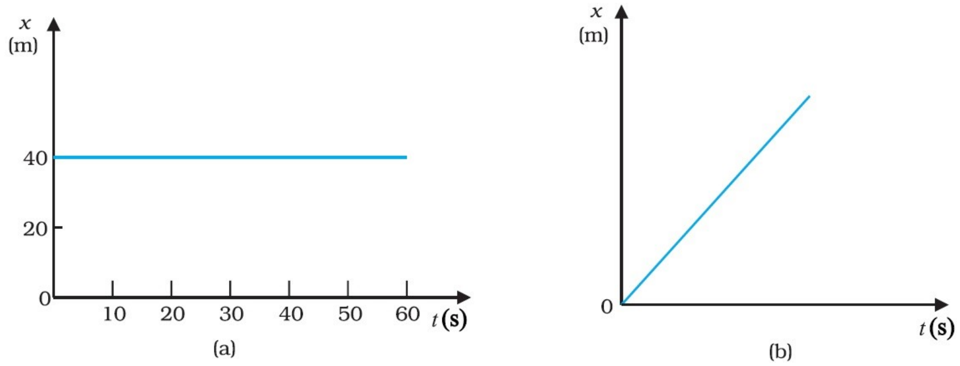

Figure 2.1: Position-time graph of (a) stationary object, and (b) an object in uniform motion.

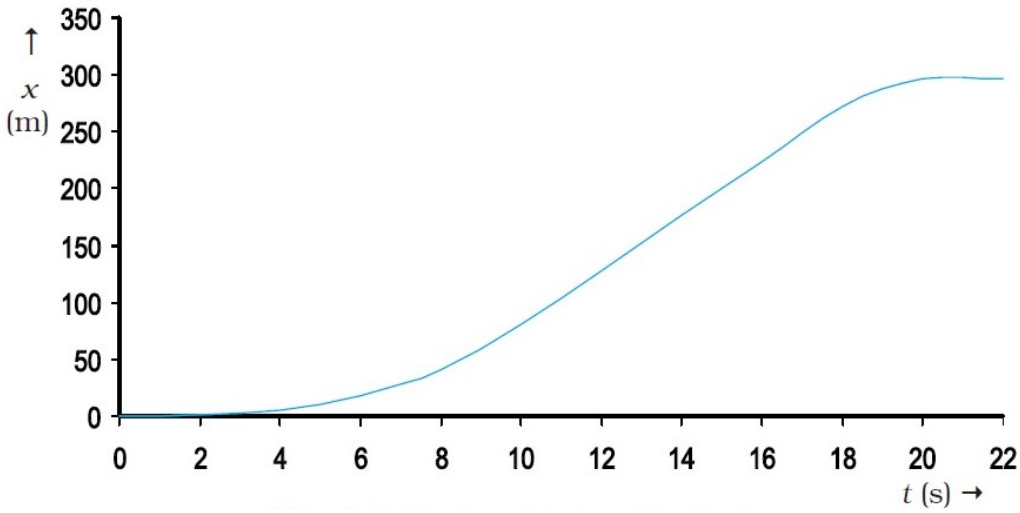

Figure 2.2: Position-time graph of a car.

3 AVERAGE VELOCITY AND AVERAGE SPEED

3.1 Average velocity

Average velocity is defined as the change in position or displacement (Δx) divided by the time intervals (Δt), in which the displacement occurs :

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \tag{3.1}$$

where x_2 and x_1 are the positions of the object at time t_2 and t_1, respectively.

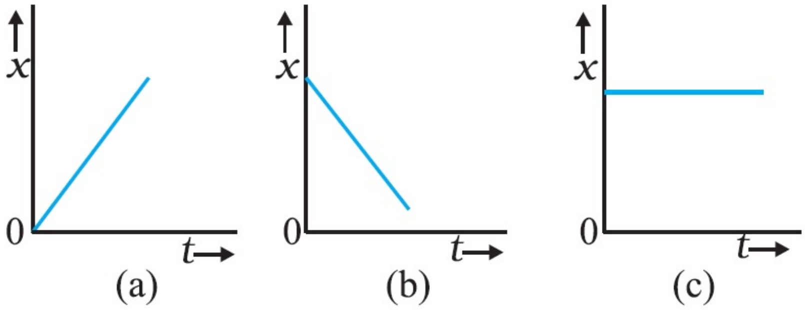

Figure 3.1: Position-time graph for an object (a) moving with positive velocity, (b) moving with negative velocity, and (c) at rest.

3.2 Average Speed

Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place :

$$Average - Speed = \frac{\text{Total Path Length}}{\text{Total time taken}}$$

4 INSTANTANEOUS VELOCITY AND SPEED

4.1 Instantaneous velocity

The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small. In other words,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \tag{4.1}$$

$$v = \frac{dx}{dt} \tag{4.2}$$

4.2 Instantaneous speed

Instantaneous speed or simply speed is the magnitude of velocity.

5 ACCELERATION

5.1 Average Acceleration

The average acceleration a over a time interval is defined as the change of velocity divided by the time interval :

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \tag{5.1}$$

where v_2 and v_1 are the instantaneous velocities of the object at time t_2 and t_1, respectively. It is the average change of velocity per unit time. The SI unit of acceleration is m s–2 .

5.2 Instantaneous acceleration

The acceleration at an instant is defined as the limit of the average acceleration as the time interval Δt becomes infinitesimally small. In other words,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \tag{5.2}$$

$$a = \frac{dv}{dt} \tag{5.3}$$

The acceleration at an instant is the slope of the tangent to the v–t curve at that instant.

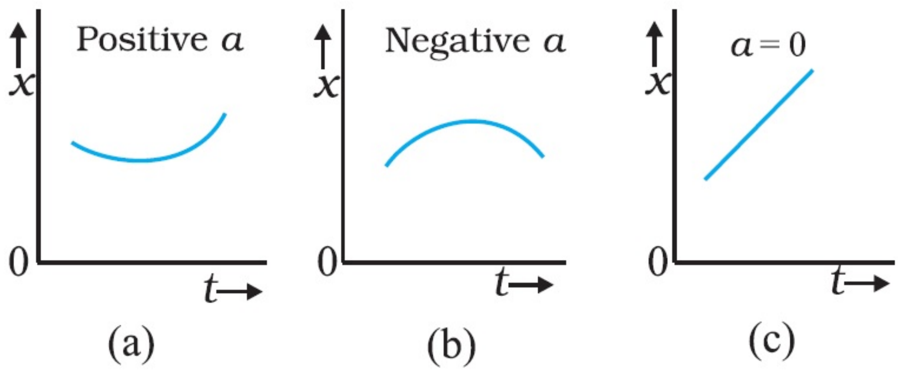

Figure 5.1: Position-time graph for motion with (a) positive acceleration; (b) negative acceleration, and (c) zero acceleration..

5.3 Velocity Time Graphs

A velocity-time graph for any moving object is that the area under the curve represents the displacement over a given time interval.
Velocity–time graph for motions with constant acceleration.

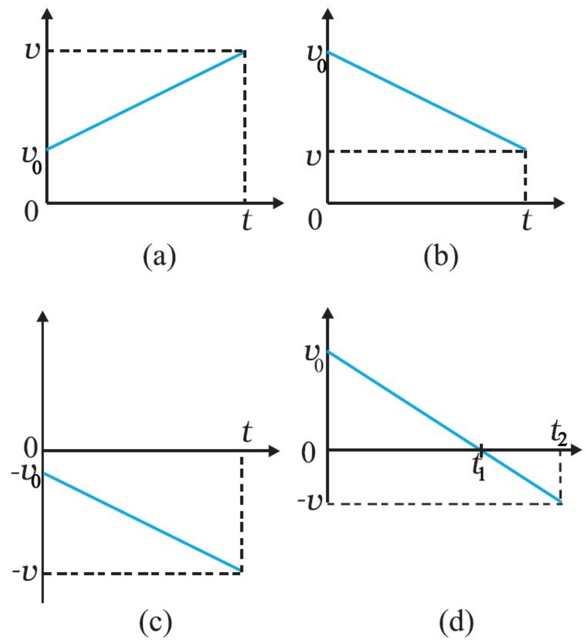

Figure 5.2: (a) Motion in positive direction with positive acceleration, (b) Motion in positive direction with negative acceleration, (c) Motion in negative direction with negative acceleration, (d) Motion of an object with negative acceleration that changes direction at time t_1. Between times 0 to t_1, its moves in positive x - direction and between t_1 and t_2 it moves in the opposite direction.

An object moving with constant velocity u. Its velocity-time graph is as shown in Fig

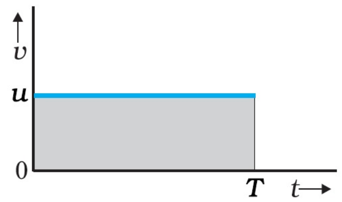

Figure 5.3: Area under v–t curve equals displacement of the object over a given time interval.

6 KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION)

- For uniformly accelerated motion, we can derive some simple equations that relate displacement (x), time taken (t), initial velocity (v_0), final velocity (v) and acceleration (a).

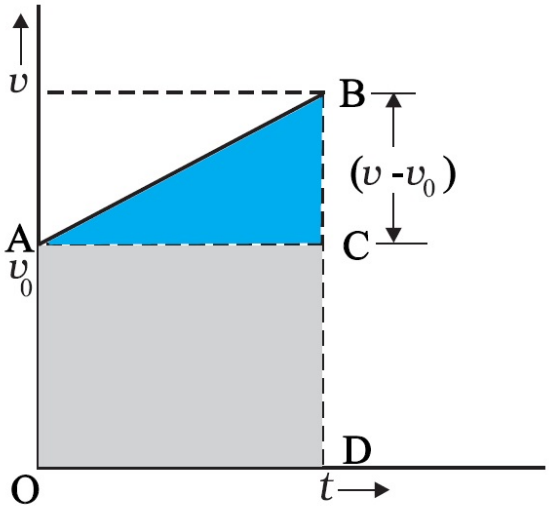

Figure 6.1: Area under v-t curve for an object with uniform acceleration.

- A relation between final and initial velocities v and v_0 of an object moving with uniform acceleration a :

$$\boxed{v = v_o + at} \tag{6.1}$$

This relation is graphically represented in Fig. 6.

- The area under this curve is :
Area between instants 0 and t = Area of triangle ABC + Area of rectangle OACD
the area under v-t curve represents the displacement. Therefore, the displacement x of the object is :

$$x = \frac{1}{2}BC \times AC + OA \times OD \tag{6.2}$$

$$x = \frac{1}{2}(v - v_o)t + v_o t \tag{6.3}$$

but

$$v - v_o = at \tag{6.4}$$

$$\boxed{x = \tfrac{1}{2}at^2 + v_o t} \tag{6.5}$$

- Equation (6.3) can also be written as

$$x = \left(\frac{v + v_o}{2}\right) t = \bar{v} t \tag{6.6}$$

Where,

$$\bar{v} = \frac{v + v_o}{2} \tag{6.7}$$

From the Equation

$$t = \frac{v - v_o}{a} \tag{6.8}$$

$$x = \bar{v} t = \left(\frac{v + v_o}{2}\right)\left(\frac{v - v_o}{a}\right) = \frac{v^2 - v_o^2}{2a} \tag{6.9}$$

$$\boxed{v^2 = v_o^2 + 2ax} \tag{6.10}$$

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First printing, 10 September 2022