Physics Supplement to my class

Learners should note that I have used symbols for geometrical figures and abbreviations through out the document.

Kinetic Theory of Gases

1 Ideal Gas Equation

Gases at low pressures and high temperatures approximately satisfy a simple relation between their pressure, temperature and volume given by

$$PV = Nk_BT \tag{1.1}$$

Where, k_B = Boltzmann Constant, N = Number of Molecules

In terms of **Moles**

$$PV = \mu RT \tag{1.2}$$

Where, μ = Number of Moles, $Nk_B = R$ = Universal Gas Constant

A gas that satisfies equation 1.2 at all pressures and temperatures is defined as an **IDEAL GAS**.

2 Pressure of an Ideal Gas

3 Most Probable, Mean and RMS Speed of Gas Molecules

3.1 Most Probable Speed

Most probable speed of the molecules of a gas is that speed which is possessed by maximum fraction of total number of molecules of the gas.

FOR EXAMPLE: If the speed of 10 molecules of a gas are 2,3,3,4,4,4,5,6,7,7,m/s, then the most probable speed is $4m/s$, as maximum fraction of total molecules possess this speed and important to note here that it's not the maximum speed of the molecules.

In the case of Ideal Gases

$$v_{mp} = \sqrt{\frac{2k_BT}{m}} \tag{3.1}$$

Where m is the mass of the molecule, k_B is Boltzmann constant and T is temperature of the gas. For a given gas , value of most probable speed increases with rise in temperature.

3.2 Mean Speed or Average Speed

It is the Average speed with which a molecule of a gas moves.

It is equal to the sum of the individual speeds of the molecules divided by the number of molecules. If $v_1, v_2, v_3, ...v_n$ are the speeds of n individual molecules, then their mean or average speed is

$$v_{av} = \frac{v_1 + v_2 + v_3 + ... + v_n}{n} \tag{3.2}$$

3.3 Root Mean Square Speed

It is defined as the square root of the mean of the squares of the random velocities of the individual molecules of a gas.

If $v_1, v_2, v_3, ...v_n$ are random velocities of n molecules of a gas, then rms speed of molecules is

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + ... + v_n^2}{n}} \tag{3.3}$$

For Ideal gases

$$v_{rms} = \sqrt{\frac{3k_B T}{m}} \tag{3.4}$$

Where, the symbols have their usual meaning.

NOTE:- From equations (3.1),(3.2), (3.3), we find that $v_{mp} : v_{av} : v_{rms} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$
v_{rms} is maximum and v_{mp} is minimum.

Here is an example to clear the above concepts

EXAMPLE :
You are given five numbers: 7, 13, 34, 69, and 91.

1. What is the average value n_{av} of these numbers?
2. What is the rms value n_{rms} of these numbers?

ANSWER:
(1) The average value can be found from

$$n_{av} = \frac{7 + 13 + 34 + 69 + 91}{5} = 42.8 \rightarrow (Answer)$$

(2) The rms value can be found from

$$n_{rms} = \sqrt{\frac{7^2 + 13^2 + 34^2 + 69^2 + 91^2}{5}} = 53.7 \rightarrow (Answer)$$

The rms value is greater than the average value because the larger numbers –being squared– are relatively more important in forming the rms value.

4 Degree of Freedom(DoF)

The number degree of freedom of dynamical system is defined as the total number of co-ordinates or independent quantities required to describe completely the position and configuration of the system.

For example: (Considering a point mass of the particle without Rotation.)

1. If a particle is moving along a straight line, say along X- axis, its position can be specified by its displacement along the X-axis. Therefore, such a particle has one translational degree of freedom.
   e.g. A bob of an oscillating simple pendulum.

2. When a particle is moving in a plane, its position can be determined by its displacements along the X-axis and Y-axis. Therefore, it has two translational degree of freedom.
   e.g. An Ant moving on the floor.

3. When a particle is moving in space, its position can be determined by its displacements along the X-axis, Y-axis and Z-axis. Therefore, it has three translational degree of freedom.
   e.g. A buzzing bee, flying birds etc.

Consider a system of two particles, each having three degree of freedom so that the number of DoF of both particles is six. If the two particles remain at a fixed distance (a definite relation)from each other then the number of coordinates required to describe the configuration of the system reduces by one.Therefore, the system has 5 DoF.

Generalising the above observation

The number of DoF (N) of a dynamical system is obtained by subtracting the number of independent relations from the total number of coordinates required to specify the position of constituent particles of the system.

i.e.

$$N = 3A - R \tag{4.1}$$

Where, A = number of particles of the system, R = number of independent relations among particles

- **For Mono atomic gases**(e.g. Neon, Argon, Helium etc.) put $A = 1, R = 0$ in eqn.(4.1) therefore $N = 3 \times 1 - 0 = 3$

- **For Di atomic gases**(e.g. H_2, O_2, N_2 etc.)
  The molecule is capable of translatory motion of its centre of mass. Therefore it has

three translational DoF. But in addition , the molecule can rotate about its centre of mass in horizontal and vertical planes. Thus a di atomic molecule also has two rotational DoF.
Also, Put A = 2 and R = 1 in eqn.(4.1), we get

$$N = 3 \times 2 - 1 = 5$$

- **For Tri atomic gases**(e.g. H_2S, SO_2 etc.)
  - **Linear Molecule** (• − − − • − − − •)
    In this case, $A = 3$ and $R = 2$(as two distances between the atoms are fixed).
    from (4.1), $N = 3 \times 3 - 2 = 7$
  - **Non-Linear Molecule** (atoms at the vertices of a △)
    In this case, $A = 3$ and $R = 3$(as three distances between the atoms are fixed).
    from (4.1), $N = 3 \times 3 - 3 = 6$

5 Determination of γ from the DoF

Suppose a poly atomic gas molecule has n DoF.
Therefore Internal energy of one gram mole of the gas is

$$U = n \times \tfrac{1}{2} k_B T \times N_A = \tfrac{n}{2} RT$$

$$\begin{aligned}
C_v &= \frac{dU}{dT} = \frac{d}{dT}\left(\frac{n}{2}RT\right) = \frac{n}{2}R \\
C_p &= C_v + R \\
C_p &= \frac{n}{2}R + R = \left(\frac{n}{2}+1\right)R \\
\gamma &= \frac{C_p}{C_v} \\
\gamma &= \frac{\left(\frac{n}{2}+1\right)R}{\frac{n}{2}R} = \frac{2}{n}\left(\frac{n}{2}+1\right)
\end{aligned}$$

$$\gamma = \left(1 + \frac{2}{n}\right) \tag{5.1}$$

This is the relation between the γ and DoF.

QUESTIONS

1. When alcohol or acetone (nail polish remover) is rubbed on your body, your body temperature decreases. Explain this effect.

2. A liquid partially fills a container. Explain why the temperature of the liquid decreases if the container is then partially evacuated. (Using this technique, one can freeze water at temperatures above $0°C$.)

3. When an automobile travels for a long distance the air pressure in the tyres increases. Why?

4. A gas storage tank has a small leak. The pressure in the tank drop more quickly if the gas is hydrogen than if it is oxygen. Why?

5. Why the land has a higher temperature than the ocean during the day but a lower temperature at night?

6. Although the velocity of air molecules is nearly $0.5 km/s$ yet the smell of scent spreads at a much slower rate. Why?

7. Why evaporation causes cooling?

8. When air is pumped into a cycle tyre the volume and pressure of the air in the tyre both are increased. What about Boyle's law in this case?

9. Explain why there is fall in temperature with altitude.

ANSWERS

1. When we pour some alcohol or acetone (nail polish remover) on our palm the particles gain energy from your palm or surroundings and evaporate causing the palm to feel cool. (See also answer 7)

2. The faster-moving atoms are the ones that escape from the liquid's surface (that is, they are the ones that evaporate). That means the average speed of the remaining atoms is decreased – and the temperature decreases as the average speed of the atoms decreases.

3. On driving an automobile for a long time, the work done against friction is converted into heat. The gas in the tyre gets heated and hence the pressure of the gas increases, because $P \propto T$

4. Rate of diffusion of a gas is inversely proportional to the square root of the density. so hydrogen leaked out more rapidly.

5. Specific Heat of water is more than land(Earth). Therefore for given heat changes in temperature of land is more than ocean.

6. The air molecules travel along a zigzag path due to frequent collision as a result their displacement per unit time is very small.

7. During evaporation fast moving molecules escape from a liquid surface so the average kinetic energy of the molecules left behind is decreased thus the temperature of the liquid is lowered.

8. When air is pumped, more molecules are pumped in. Boyle's law is applicable only when number of molecules remains constant. So Boyle's law is not obeyed.

9. As the molecules move higher their potential energy increases and hence kinetic energy decreases and hence temperature decreases. Also at greater heights, more volume is available. The gas expends and hence cooling occurs.

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